Pair of Straight lines

MD. MOHIUDDIN

LECTURER, CUMILLA university

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**Pair of straight lines**

**Pair of straight lines:** A pair of straight lines is the locus of a point whose coordinates satisfy a second degree equation . A collection of combined two straight lines is called a pair of straight lines.

**Homogeneous equation:** An equation, in which degree of each term is equal, is called a homogeneous equation. Such as  is a homogeneous equation of degree or order 2 because degree of its each term is two. It is noted that homogeneous equation always represents straight lines passing through the origin.

**Non-homogeneous equation:** An equation, in which degree of each term is not equal, is called a non-homogeneous equation. Such as  is a non- homogeneous equation of degree or order 2.

**θ**

**Theorem-01:** Prove that a homogeneous equation of the second degree always represents a pair of

straight lines through the origin.

**Proof:** The homogeneous equation of second degree is,



Dividing both sides of (1) by  and  (if ), we have



This represents a quadratic equation in . Let  and  be the roots of this equation.

Sum of the roots is



and product of the roots is

.

The equation (2) must be equivalent to



The two lines represented by (2) i.e. (1) are given

, and 

i.e. , and .

Which pass through the origin.

Thus, the homogeneous quadratic equation  always represents a pair of straight lines, real or imaginary, through the origin. (**Proved**).

**Alternatively**, Multiplying both sides of (1) by  (if ), we have









,

which represent two straight lines, real or imaginary through the origin. (**Proved**).

**Theorem-02:** Find the angle between the straight lines represented by the homogeneous equation

.

**Proof:** Given homogeneous equation of second degree is,



Suppose, and  be the lines represented by (1).

So, 





This equation is same as to the equation (1), so the ratios of the coefficient of like terms are equal.

Now comparing the coefficients



From 2nd and 3rd parts, we get 

From 1nd and 3rd parts, we get 

If  be the angle between two straight lines represented by the given equation then











 **(Proved).**

**Theorem-03:** Find the condition that general equation of second degree  may represents two straight lines.

**Proof:** Given general equation of second degree is,



The above equation can be written as,



Solving we get,













Equation (1) represents two straight lines if it is possible to factorize the left hand side of (1) as a product of two linear factors.

It will be done if the quantity of under the square root sign in the equation (2) be a perfect square.

That means  must be perfect square. We know, be perfect square if the roots of the equation are equal.

Now, 







Roots of the above quadratic equation in y are equal if the discriminant of the equation.

Here,



















This is the required condition to represents the straight lines. **(Proved).**

**Theorem-04:** Find the equation of the bisectors of the angles between the straight lines represented by the

homogeneous equation .

**Proof:** Given homogeneous equation of second degree is,



If  and  be the roots of this equation, then the lines represented by the equation (1) are



and 

Sum of the roots is



and product of the roots is

.

The equations of the required bisectors are























This is the required equation of the bisector.

**Note:**

* Angle between the lines represented by the equation is calculated by formula .
* Lines be perpendicular if 
* Lines be parallel if 
* Lines represented by homogeneous or non-homogeneous equation are real if.
* Lines represented by homogeneous or non-homogeneous equation are imaginary if.
* The equation of the bisectors of an angle produced by the pair of straight line represented by the homogeneous equation  is .
* The equation of the bisectors of an angle produced by the pair of straight line represented by the equation is , where is the intersection point of those lines.
* The intersecting point  of the straight lines represented by the equation



Let 

Then set 



Solving Eq. (1) & Eq. (2) we get,



This is the point of intersection of the straight lines.

**Problem-01:** Find the angle between the lines represented by the equation.Also find the equations of the straight lines and equation of the bisector’s of the angle.

**Solution: 1st part:** Given that,



The general equation of second degree in homogeneous form is,



Comparing Eq. (1) & Eq. (2) we have,

.

Let  be the angle between the lines. So, the angle is,











Therefore, the angle between the lines is, .

**2nd part:** The given equation can be written as,













Taking positive sign we get,







Again taking negative sign we get,







Therefore, and  are the required straight lines passing through the origin.

**3rd part:** The equation of bisector’s of the angle produced by the straight lines is,











This is the required equation of the bisector’s.

**H.W:** Find the angle between the lines represented by the following equations. Also find the equations of

the straight lines and equation of the bisector’s of the angle.

1. .
2. .
3. .
4. .
5. .
6. .
7. .
8. .
9. .

**Problem-02:** Show that represents pair of straight lines. Also find their

equations, the point of intersection, the angle and the equation of the bisector’s of angle.

**Solution: 1st part:** Given that,



The general equation of second degree is,



Comparing Eq. (1) & Eq. (2) we have,



Now, 















Since, so the given equation represents a pair of straight lines. (***Showed***)

**2nd part:** The given equation can be written as the following quadratic equation in *x*,





















Taking positive sign we get,













Taking negative sign we get,













Therefore, required equations of the straight lines  and .

**3rd part:**  Suppose, 

Differentiating the function with respect to *x* and *y* partially and equating with zero, we get



And



Solving Eq. (3) &Eq.(4) we get the point of intersection of lines represented by the given equation.

Using cross multiplication method on Eq. (3) & Eq.(4)







Therefore, the coordinates of the point of intersection is *i.e.*.

**4th part:** If be the angle between the lines then,















Since, the angle is so the lines are perpendicular.

**5th part:** The equation of the bisector’s is,

















**(As desired).**

**Problem-03:** Show that represents pair of straight lines. Also find their

equations, the point of intersection, the angle and the equation of the bisector’s of angle.

**Solution: 1st part:** Given that,



The general equation of second degree is,



Comparing Eq. (1) & Eq. (2) we have,



Now, 











Since,  so the given equation represents a pair of straight lines. (***Showed***)

**2nd part:** The given equation can be written as the following quadratic equation in *x*,

















Taking positive sign we get,











Taking negative sign we get,











Therefore, required equations of the straight lines  and .

**3rd part:**  Suppose,  be the point of intersection of the lines.











Therefore, the point of intersection is*,*.

**4th part:** If  be the angle between the lines then,



















**5th part:** The equation of the bisector’s is,

















**(As desired).**

**Problem-04:** Show that represents pair of straight lines. Also find their

equations and the angle.

**Solution: 1st part:** Given that,



The general equation of second degree is,



Comparing Eq. (1) & Eq. (2) we have,



Now, 







Since,  so the given equation represents a pair of straight lines. (***Showed***)

**2nd part:** The given equation can be written as the following quadratic equation in *x*,















Taking positive sign we get,











Taking negative sign we get,











Therefore, required equations of the straight lines  and .

**3rd part:** If  be the angle between the lines then,











**(As desired).**

**H.W:** Show that the following equations represent pair of straight lines. Also find their equations, the point of intersection, the angle and the equation of the bisector’s of angle.

1. 
2. 
3. 
4. 
5. 

**Problem-05:** For what value of  the equation represents a pair of

straight lines.

**Solution:** Given that,



The general equation of second degree is,



Comparing Eq. (1) & Eq. (2) we have,



Here, the given equation represents a pair of straight lines if .

Now, 















This is the required value of.**(Ans)**

**Problem-06:** For what value of  the equation represents a pair of

straight lines.

**Solution:** Given that,



The general equation of second degree is,



Comparing Eq. (1) & Eq. (2) we have,



Here, the given equation represents a pair of straight lines if .

Now, 





























These are the required values of.**(Ans)**

**H.W:**

1. For what value of the equation represents a pair of straight lines.
2. For what value of  the equation represents a pair of straight lines.
3. For what value of  the equation represents a pair of straight lines.
4. For what value of  the equation represents a pair of straight lines.

**Question-01:** Describe various conditions of general equation of second degree.

**Answer:** The general equation of second degree is,

This will represent the followings,

1. A pair of straight lines if the determinant,

Two parallel lines if

Two perpendicular lines if

1. A circle if
2. A parabola if
3. An ellipse if
4. A hyperbola if
5. A rectangular hyperbola if

**Problem-07:** Test the nature of the equation and also find its

centre.

***Solution: 1st part:*** Given that,

Also the general equation of second degree is,

Comparing and we have,

Now,

.

Since, so the given equation represents a conic.

Again,

And,

Since, so the given equation represents a rectangular hyperbola.

***2nd part:*** Let,

And

The centre of the conic is the intersection of two lines,

Solving and we have,

Hence the centre is at (**As desired )**

**Problem-08:** Test the nature of the equation.

***Solution:*** Given that,

Also the general equation of second degree is,

Comparing and we have,

Now,

.

Since, so the given equation represents a conic.

Again,

Since, so the given equation represents a parabola.(**As desired )**

**H.W:**

Test the nature of the following equations and find its centre.

**Reduction of equation to a standard form**

**Problem-09:** Reduce the equation to the standard form.

**Solution:** Given that,



The general equation of second degree is,



Comparing Eq. (1) & Eq. (2) we get,



Now,









and



Since  and . So the equation represents an ellipse.

Let, be the centre of conic.



and 

Therefore, the coordinates of centre is .

Therefore, the equation of the conic referred to centre as origin is,



where,



So the equation becomes,



When the xy term is removed by the rotation of axes then the reduced equation is,



Then by invariants we have



and, 





We know,











Solving equations and we have  and 

The equation becomes 



This is required equations.(**As desired)**

**Problem-10:** Reduce the equation to the standard form.

**Solution:** Given that,



The general equation of 2nd degree is,



Comparing Eq. & Eq.we get,



Now,



and 

Since, and . So the equation represents hyperbola.

Let, be the centre of conic.



and 

Therefore, the coordinates of centre is .

Therefore, the equation of the conic referred to centre as origin is,



where,



So the equation becomes,



When the xy term is removed by the rotation of axes then the reduced equation is



Then by invariants we have



and, 





We know,











Solving equations and we have  and 

The equation becomes,



This is required equation. (**As desired)**

**Problem-05:** Reduce the equation to the standard form.

**Solution:** Given that,



The general equation of 2nd degree is,



Comparing Eq. &Eq.we get,



Now,



and



Since, and . So the equation representsa parabola.

From given equation we have,









The lines  and are perpendicular if

.

i.e. 





Putting the value of  in (3) we get

















where, 

That is the standard form of Parabola.

**H.W:**

Reduce the following equations to the standard forms

1.  Ans: 
2.  Ans: 
3.  Ans: 
4.  Ans: 
5.  Ans: 